Chapter 1

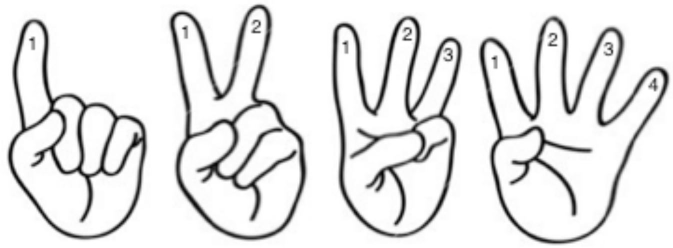
The Number Line to 10

One of the biggest mathematical mistakes that early elementary school teachers make is expanding the set of numbers that their students work with too quickly. Many teachers defend their choices, stating that their students enter kindergarten already knowing how to count to 10. In turn, they "challenge" them by training the class to count to 20, 50, and even 100. This notion that *Bigger is more challenging* is inaccurate.

Just as the widest, sturdiest foundations create the tallest towers, the broadest, deepest foundations develop the strongest math students. While the idea of kindergarteners and first graders counting to very high numbers sounds impressive in faculty room and parent conference discussions, little mathematical purpose is served. Memorizing the sequence of numbers from zero to ten represents mastery of the number line no more than reciting the order of U.S. Presidents from Washington to Obama signifies a deep understanding of American history.

It's common for children to leave kindergarten being able to count to 100, but very few leave with a deep understanding of numbers to ten. Just as history becomes meaningful when students know details and "back stories" between major events, a child masters numbers to ten when they can fluently transition forward and backward from any position on the number line *and* understand all of its equivalent forms.

Americans normally represent 1 by holding up their index finger; 2 by holding up their index and middle fingers; 3 by extending their index, middle and ring fingers; and 4 by holding up every finger except their thumb. This creates a symbolic number line to 4 working left to right across the counter's vision.



There is a logical breakdown, however, once the thumb is extended. For the hand to represent a number line, the fifth digit should extend to the right of the pinky, like a sixth finger appendage. But, the American finger counting tradition extends the fifth finger in what would logically be the zero placeholder.

The first step to understanding the number line is for students to visually recognize the numeral and create a word association with it. The numeral **1** symbolizes one object and is pronounced *One*. Teachers can add greater intentionality to this concept by using the *Laranang Glove* to direct choral class counting. Laranang Glove counting (named after a pre-K student who excelled at this type of counting) involves a teacher wearing a glove on their right hand with the numbers 1, 2, 3, 4, and 5 written on their pinky, ring, middle, index, and thumb fingers, respectively.[1] Then, by extending their pinky, ring, middle, index, and thumb digits in order, students see a number line form from left to right across their vision, because the numeral five appears in the logical position.

[1] To the best of my knowledge, Robin Ramos, a brilliant elementary Math trainer from Los Angeles, invented glove counting.

Looking at the teacher's right hand from the child's perspective.

When students are ready to count to six, the teacher puts a second glove on their left hand. The 6, 7, 8, 9, and 10 is written on their left thumb, index, middle, ring, and pinky fingers, respectively.

Looking at the teacher's left and right hands from the child's perspective.

As the teacher extends their fingers in order, students see a number and say it, e.g. they see two fingers and the numeral 2 is written on the second finger that their teacher extends, so they say "Two." When the teacher extends their third finger, the students see the numeral 3 and say "Three." To build an understanding of the number line to 3, the teacher will extend and fold down their fingers, forcing students to concentrate on changes of direction. These sequences can be planned, but certainly not scripted. A teacher needs to react to student understanding and misunderstandings, staying with shaky transitions until they are mastered.

Once students can visually recognize their numbers, they are ready for a new challenge and the process is repeated with the gloves off or turned in the other direction, so that they can no longer see numbers. Consider the following sequence:

1 2 1 2 3 2 3 2 1

Students see one finger and say “one”, two fingers and say “two”, etc. Once students can comfortably transition on the number line between 0 and 3, they are ready to work with the number line to four. After four, then five and so on to ten. It is better for a child to be able to count to a low number and comfortably change directions than it is for them to count linearly to a high number, without being able to change directions. The former builds a tower with a strong foundation of comfort and understanding. The latter stretches too high on that same tower without a sturdy foundation.

Developing a mastery of the number line, of course, entails more than stating the number of objects (in this case fingers) that they see. They need to internalize the transitions mentally. Dr. Yoram Sagher, professor of mathematics at Florida Atlantic University, developed a drill called “Happy Fingers” to introduce this new level of abstraction. Happy Fingers is led by three simple hand directions. When a teacher points up, the students count one unit up on the number line; when the teacher holds their hand sideways, the students stop counting; and when the teacher points down, the students count one unit down on the number line. This forces students to think in two directions on the number line. It also empowers teachers to emphasize important and/or difficult transitions. Again, it is better for students to count to a low number while comfortably changing directions than it is to linearly count to a high number without being able to make the transitions. Challenges can be added by directing students to “Whisper” or “Think” intervals on the number line.

Say 5 Counting

Students build an understanding of the number line through teacher-directed counting. Once students can comfortably change directions within a set of numbers, then they have mastered the number line within that set and are prepared to deepen their number sense through equivalent counting forms. The equivalent counting form with the most utility for numbers 1 to 10 is “Say 5” counting. Recognizing that an open hand is equal to 5 is the first major number grouping discovery that children make. It helps them access bigger numbers more quickly and develops efficient strategies for adding and subtracting by 5. In the following example, notice how the complexity grows.

1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	5 & 1	5 & 2	5 & 3	5 & 4	5 & 5
1	2	3	4	5	5 & 1	7	5 & 3	9	5 & 5
1	2	3	4	5	6	5 & 2	8	5 & 4	10

The idea of *Say* 5 counting might be introduced with the following dialogue. (Teachers could direct students to count using the Laranang Glove or Happy Fingers.)

T: (Direct students to count to 10.)
S: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
T: Today, I’m going to teach you a new way to count to 10. First, count with me to 5. (Direct students to count to 5.)
Students count as teacher extends fingers.

1 2 3 4 5

T: What number comes next?
S: 6.
T: Yes. But we’re now going to count the *Say 5* way. Instead of saying 6, you’ll say *Five and 1*.

T: What would come after 5 and 1?

S: 5 and 2.
T: Next?

S: 5 and 3.
T: You get the hang of it! Let's count to 10 again. This time, when you pass 5, count the *Say 5* way.
S: 1, 2, 3, 4, 5, 5 and 1, 5 and 2, 5 and 3, 5 and 4, 5 and 5.

After students are comfortable counting the *Say 5* way, they are ready to shift back and forth between equivalent forms. The following script might take place.

T: Count to 10 the *Say 5* way. When I raise my hand, stop.
S: 1, 2, 3, 4, 5, 5 and 1, 5 and 2.
T: (Raise hand.) What number is 5 and 2?
S: 7.
T: Continue.
S: 5 and 3, 5 and 4, 5 and 5.
T: (Raise hand.) What number is 5 and 5?
S: 10.
T: Count down, starting at 5 and 5.
S: 5 and 5, 5 and 4, 5 and 3.
T: (Raise hand.) What number is 5 and 3?
S: 8.
T: Continue.
S: 5 and 2, 5 and 1.
T: (Raise hand.) What number is 5 and 1?
S: 6.
T: Continue.
S: 5, 4, 3, 2, 1.

An advanced drill would lead the students to alternate between regular and *Say 5* counting.

T: Count to 10 again. This time, alternate between Say 5 counting and the regular way.

Listen to me. 1, 2, 3, 4, 5, 5 and 1, 7, 5 and 3. Now, you give it a try.

S: 1, 2, 3, 4, 5, 5 and 1, 7, 5 and 3, 9, 5 and 5.

T: What number is the same as 5 and 5?

S: 10.

T: Starting at 10, count backwards, alternating between Say 5 and regular counting.

S: 10, 5 and 4, 8, 5 and 2, 6, 5, 4, 3, 2, 1.