Preface

Two Forms of Teacher-led Counting

1. Skip Counting by Changing Directions on the Number Line

A deep understanding of the number line is essential to understanding and succeeding in elementary mathematics. Skip counting while changing directions is the most efficient method I know of to gain a mastery of the number line, because it demands that students comfortably shift directions, incorporating adding, subtracting, multiplying, and dividing into one focused exercise.

Traditionally, counting in elementary school classrooms consists of reciting numbers in ascending and/or descending order, e.g. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1. Mastery of this linear form of counting entails memorizing a script. As a result, students internalize the start and endpoints, and the sequence of numbers in between. However, when asked to make a transition within the sequence, e.g. *What's 1 more than 5?,* the student who has memorized the script as a song or poem, might need to return to the starting point to access the transition, similarly to the way they might need to access the second to last line of the *Pledge to Allegiance* by starting with the pledge's first line.

Linear counting with a defined start and endpoint assigns an equal importance to each transition, because each number in the sequence is spoken an equal amount. All skip counting transitions, however, are not equal in difficulty and thus shouldn't be practiced the same amount of time. When learning to play a song on guitar, a beginner will spend more time practicing a difficult chord transition, e.g. D to C, than they will G to e minor, because mastering the dexterity and rhythm of the former requires far more

practice than achieving it with the latter. To illuminate this point, let's look at the example of skip counting by threes to 30.

0 3 6 9 12 15 18 21 24 27 30

The 0 to 3, 3 to 6 and 6 to 9 transitions tend to be relatively simple for second and third grade students, as are the jumps from 12 to 15, 15 to 18, 21 to 24, and 24 to 27. The 9 to 12, 18 to 21, and 27 to 30 transitions, however, are much harder, because these shifts cross a ten. Therefore, to master skip counting by threes the difficult transitions should be emphasized. Consider the following sequence:

0, 3, 6, 9, 12, 9, 12, 9, 12, 15, 18, 21, 18, 21, 24, 27, 30, 27, 30, 27, 24, 21, 18, 21, 18, 21, 18, 15, 12, 9, 12, 9, 6, 3, 0

Students, of course, will not practice counting this way unless they are directed to do so in class. A teacher can easily manage this practice by cuing their students to count up when they point up, to stop when they hold their hand sideways, and to count down when they point down. Initially, students often struggle, because it requires them to focus in two directions on the number line and they're not used to sustaining this high level of concentration. On the first day of school, a teacher can usually hold their classes' collective focus for about 30 seconds before their students become exhausted by the exercise. With routine practice, however, student stamina improves and holding a class' focus for several minutes becomes possible. After a class is collectively fluent, the drill takes on a rhythmic form that becomes contagious to participate in. Best of all, once their endurance is built, children's latent ability to focus permeates not only to their Math lessons, but their other studies as well.

2. Counting with Equivalency on the Number Line

Students build a mastery of the number line through counting different units while changing directions. Once students can count fluently on the number

line by a given unit, they can deepen their understanding of mathematical concepts by incorporating equivalency into number line counting through simple to complex linear progressions. The following dialogue might work as a fluency drill for a first grade class that has learned units of ten within 100 at the concrete level, and is working towards internalizing unit versus standard form at the abstract level:

Teacher: Count to 10 starting at zero.
Students: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
T: Count to 10 tens starting at zero tens.
S: 0 tens, 1 ten, 2 tens, 3 tens, 4 tens, 5 tens, 6 tens, 7 tens, 8 tens, 9 tens, 10 tens.
T: Let's count to 10 tens again. This time, when I raise my hand I want you to stop. Go.
S: 0 tens, 1 ten, 2 tens, 3 tens.
T: (Raise hand.) Say 3 tens in standard form.
S: 30.
T: Continue.
S: 4 tens, 5 tens, 6 tens, 7 tens.
T: (Raise hand.) Say 7 tens in standard form.
S: 70.
T: Continue.
S: 8 tens, 9 tens, 10 tens.
T: (Raise hand.) Say 10 tens in standard form.
S: 100.
T: Count backwards in standard form, starting at 100.
S: 100, 90, 80, 70, 60.
T: (Raise hand.) How many tens is 60?
S: 6 tens.
T: Continue.
S: 50, 40, 30, 20.
T: (Raise hand.) How many tens is 20?
S: 2 tens.
T: Continue.
S: 10, 0.

Knowing when to stop the progressions and signal for an equivalent form is a rhythm and balance that teachers need to develop. This book contains hundreds of sequences, but these stops and sequences are not written into them. However, the "correct time" to deliver this stage in the sequence is

after the early lines of the sequence are understood, but before the forms alternate. The line in the sequence below marks when the above dialogue would take place.

1	2	3	4	5	6	7	8	9	10
1 Ten	2 Tens	3 Tens	4 Tens	5 Tens	6 Tens	7 Tens	8 Tens	9 Tens	10 Tens
10	20	30	40	50	60	70	80	90	100
1 Ten	20	3 Tens	40	5 Tens	60	7 Tens	80	9 tens	100
10	2 Tens	30	4 Tens	50	6 Tens	70	8 Tens	90	10 Tens

In many cases the complexity symbolized by the line is sufficient challenge for a class to develop fluency. The exercise, however, can be extended by directing students to alternate between unit and standard form:

T: Count by 1 tens to 10 tens, alternating between unit and standard form.
S: 1 ten, 20, 3 tens, 40, 5 tens, 60, 7 tens, 80, 9 tens, 100.
T: How many tens is 100?
S: 10 tens.
T: Let's count backwards, alternating again, but starting with 10 tens.
S: 10 tens, 90, 8 tens, 70, 6 tens, 50, 4 tens, 30, 2 tens, 10.

A middle grades example might follow this progression:

0	1	2	3	4	5	6	7	8
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$	$\frac{5}{4}$	$\frac{6}{4}$	$\frac{7}{4}$	$\frac{8}{4}$
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{6}{4}$	$\frac{7}{4}$	2
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{2}{4}$	$1\frac{3}{4}$	2
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2

Notice how the complexity grows with each passing line. Like a ladder, a child needs to reach each rung before reaching the top. Start too complex

and students will be frustrated and confused. However, build a solid foundation in which students master before moving on and students can reach the top. The process of ascending these five steps might take several minutes or several days. It all depends on the students' fluency with the topic. Teachers should always begin at a level that all students feel confident answering correctly and pull (not push) the class to the highest level that they will collectively feel challenged, but not defeated. The above exercise might be delivered as follows:

T: Count to 8 starting at zero. (Write as students count.)
S: 0, 1, 2, 3, 4, 5, 6, 7, 8.
T: Count to 8 fourths starting at zero fourths. (Write as students count.)
S: 0 fourths, 1 fourth, 2 fourths, 3 fourths, 4 fourths, 5 fourths, 6 fourths, 7 fourths, 8 fourths.
T: Which fraction is equal to 1 whole?
S: 4 fourths.
T: (Cross out $\frac{4}{4}$ and write 1 beneath it.) Which fraction is equal to 2 wholes?
S: 8 fourths.
T: Cross out $\frac{8}{4}$ and write 2 beneath it.
T: (Move to the side of the board so that the number line is not in the center of the students' vision.) Let's count to 8 fourths again. This time, say the whole numbers. Try not to look at the board.
S: 0 fourths, 1 fourth, 2 fourths, 3 fourths, 1, 5 fourths, 6 fourths, 7 fourths, 2.[1]

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[1] At this juncture, teachers have a choice at how they want to build the ladder. An alternative script:

T: Let's count to 8 fourths again. This time, when I raise my hand I want you to stop. Go.
S: 0 fourths, 1 fourth, 2 fourths, 3 fourths, 4 fourths, 5 fourths.
T: (Raise hand.) Say 5 fourths as a mixed number.
S: 1 and 1 fourth.
T: Continue.
S: 6 fourths, 7 fourths, 2.
T: (Raise hand.) How many fourths is 2?
S: 8 fourths.
T: Count backwards by fourths starting at 8 fourths.
S: 8 fourths, 7 fourths.

T: (Point at $\frac{5}{4}$.) Say 5 fourths as a mixed number.

S: 1 and 1 fourth.

T: (Beneath $\frac{5}{4}$, write $1\frac{1}{4}$.)

Repeat process for $\frac{6}{4}$ and $\frac{7}{4}$.

T: (Move to the side of the board so that the number line is not in the center of the students' vision.) Let's count to 8 fourths again. This time, say the whole numbers and mixed numbers. Try not to look at the board.

S: 0 fourths, 1 fourth, 2 fourths, 3 fourths, 1, 1 and 1 fourth, 1 and 2 fourths, 1 and 3 fourths, 2.

T: (Point at $\frac{2}{4}$.) Say 2 fourths simplified.

S: 1 half.

Repeat process for $1\frac{2}{4}$.

T: (Move to the side of the board so that the number line is not in the center of the students' vision.) Let's count to 8 fourths again. This time, say the simplified fractions. Try not to look at the board.

S: 0 fourths, 1 fourth, 1 half, 3 fourths, 1, 1 and 1 fourth, 1 and 1 half, 1 and 3 fourths, 2.

For each equivalent form counting drill, a similar script can be used to build the ladder. In addition, complexity can be remediated by only counting forward and extended by counting backwards and/or changing directions.

T: (Raise hand.) Say 7 fourths as a mixed number.

S: 1 and 3 fourths.

T: Continue.

S: 6 fourths, 5 fourths, 1, 3 fourths, 2 fourths, 1 fourth, 0 fourths.

How to Read and Use this Book

While there is no hard-fast rule on how to deliver any of these counting exercises, there are a few things to keep in mind:

- Leading choral counting exercises tends to be most productive when students are refreshed and prepared to concentrate, e.g. the beginning of Math class.

- Counting drills on a given topic should always follow a concept development. The purpose of counting drills is to build fluency with previous lessons **not** teach concepts.

- When only seeing digits, the numbers should be read with compound units, e.g. 1200 is one thousand two hundred, **not** 12 hundred.

- In some sequences, I switched from digit to word form. This was to make the counting drills easier to read.

- Counting more than 12 multiples of any unit tends to be tiring for students and inefficient use of classroom time. Most drills consist of no more than ten multiples.

- Counting exercises should stimulate many other hybrid drills.

- I welcome readers to email me any questions that arise:
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